Test 3

– Knowledge

1/ $L_1: \vec{m}_1 = [-2, 4, 9]$ $L_2: \vec{m}_2 = [-2, 4, +9]$ ✓

3 $\cos\theta = \frac{\vec{m}_1 \cdot \vec{m}_2}{|\vec{m}_1||\vec{m}_2|}$ ✓ $= \frac{4+16+81}{\sqrt{101}\sqrt{101}} = \frac{+101}{101}$ ✓

4 $\theta \doteq 10°$ ✓ or (they are $\parallel$ $\therefore \theta =$

2/ $\left[\begin{array}{ccc|c} 5 & 4 & 3 & 2 \\ 3 & 2 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 2 & 2 & 2 \\ 3 & 2 & 1 & 0 \end{array}\right] R_1 - R_2$

5 $\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{array}\right] R_1 \div 2 \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{array}\right] 3R_1 - R_2$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{array}\right] R_1 - R_2$

or $5x+4y+3z=2$
$6x+4y+2z=0$
$-x+z=2$

then let $z=t$
$x-t=-2$
$y+2t=3$

so $\boxed{x=-2+t,\ y=3-2t,\ z=t}$

OR $5x+4y+3z=2 \longrightarrow 5x+4y+3z=2$
$3x+2y+z=0 \xrightarrow{\times 3} 9x+6y+3z=0$
$4x+2y=-2$
$y=-2x-1$

let $x=t$, $y=-2t-1$

$3t+2(-2t-1)+z=0$
$z=2+t$

$\boxed{x=t,\ y=-2t-1,\ z=2+t}$

Test 3

(2)

3/ (a) L_1: $x = 3-3t$, $y = 8+5t$, $z = \cancel{4}$

π_1: $7(3-3t) - 2(8+5t) + \cancel{4} - 71 = 0$

$21 - 21t - 16 - 10t + \cancel{4} - 71 = 0$

(7) $-3\cancel{0}t = 6\cancel{2}$

$t = -2$

$$\boxed{x = 9,\ y = -2,\ z = 4}$$

(b) π_2: $3-3t+2(8+5t)+7(4-t)-28=0$

$3-3t+16+10t+28-7t-28=0$

$0t = -19$ FALSE

$\therefore$ no sol'n (Line is $\parallel$ to plane)

4/ $3x+2y-z-4=0$ ①

$2x+4y+5z-3=0$ ②

$x-y+2z-7=0$ ③

$3x+2y-z-4=0$ ①

$3\times$③ $3x-3y+6z-21=0$ ④

①−④: $5y-7z+17=0$ ⑤

$5y-7z=-17$ ⑤

$2x+4y+5z=3$

$2\times$③: $2x-2y+4z=14$

$6y+z=-11$ ⑦

$7\times$⑦: $42y+7z=-77$ ⑧

⑤+⑧: $47y=-94$

$y=-2$

sub. into ⑦

$z=-11+12=1$

sub. y + z into ③

$x+2+2-7=0$

$x=3$

$\therefore$ intersection point is $(3,-2,1)$

Test 3 ③

Application

5/ (a) $y=8$, $t=2$ (b) $y=8$, $z=-3$

④ $\therefore \boxed{x=6}$ $t=2$ $\boxed{n=1}$

$-3=-n-2$

6/ $\vec{n}_1 = [2,3,-4]$, $\vec{n}_2 = [0,5,12]$ ✓

⑧ $\vec{n}_3 = \vec{n}_1 \times \vec{n}_2 = [56,-24,+10]$ ✓ ✓

2	3	-4	2
0	5	12	0
10	56	-24	

so π_3: $56x-24y+10z+D=0$ ✓

$A(5,-4,1)$

$56(5)-24(-4)+10(1)+D=0$ ✓

$D=-280-96-10=-386$

$56x-24y+10z-386=0$

$\boxed{28x-12y+5z-193=0}$ ✓

7/ $\vec{v}=[-2,2,+1]$ is on both planes ← NOTE SYMMETRIC EQ not in proper form

⑨ $C(11,-2,+3)$ ✓

so, for π_1: $\vec{AC}=[10,-3,0]$ ✓

so $\vec{n}_1 = \vec{v} \times \vec{AC} = [3,10,-14]$

-2	2	1	-2
10	-3	0	10
6-20 =-14	3	10	

π_1: $3x+10y-14z+D=0$ ✓

$3(1)+10(2)-14(3)+D=0$

$D=19$ ✓

π_1: $\boxed{3x+10y-14z+19=0}$

Test 3

$\overrightarrow{BC} = [10, -1, 2]$

(4)

7) (cont.) π_2: $\vec{n}_2 = \vec{v} \times \overrightarrow{BC} = [5, 14, -18]$ ✓

-2	2	1	-2
10	-1	2	10

$2-20 = -18 \mid 5 \mid 14$

$\pi_2: 5x + 14y - 18z + D = 0$ ✓

$5(17) + 14(\cancel{2}) - 18(\cancel{3}) + D = 0$ ✓

$D = +\cancel{8}3$

$\pi_2: \boxed{5x + 14y - 18z \cancel{+83} = 0}$

THINKING

9/ $\vec{m}_1 = [2, -3, -3]$ ✓, $B(-1, 1, 2)$ ✓

so $\vec{m}_2 = \overrightarrow{AB} = [-4, 2, 1]$ ✓

(7) $\vec{n} = \vec{m}_1 \times \vec{m}_2 = [3, 10, -8]$ ✓

2	-3	-3	2
-4	2	1	-4

$4-12 = -8 \mid -3+6 = 3 \mid 12-2 = 10$

$\pi: 3x + 10y - 8z + D = 0$ ✓

$3(3) + 10(-1) - 8(1) + D = 0$ ✓

$D = 9$

$\boxed{3x + 10y - 8z + 9 = 0}$ ✓

COMMUNICATION

10/ (a) $\vec{n}_1 = [7, -5, 4]$

$\vec{n}_2 = [-1, 3, 5]$

$\vec{n}_3 = [5, 1, 14]$

12

$\vec{n}_1 \cdot \vec{n}_2 \times \vec{n}_3$ ✓

$= [7, -5, 4] \cdot [37, 39, -16]$

$= 259 - 195 - 64$ ✓

$= 0$

$\vec{n}_2 \times \vec{n}_3 = [37, 39, -16]$

-1	3	5	-1
5	1	14	5

$-1-15 = -16 \mid 42-5 = 37 \mid 25+1 = 39$

✓ $\vec{n}_1 = \vec{n}_3 - 2\vec{n}_2$ ✓

$EQ_3 - 2EQ_2 = 5x + y + 14z - 20 - 2(-x + 3y + 5z - 2)$

$= 7x - 5y + 4z - 16$ ✓

$\neq EQ_1$ ∴ 3 planes meet in $\triangle$ prism